Partial Differential Equations
Spring 2019
Course Document[1]

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Course Aims: The aim of this course is to provide an introduction to the theory of partial differential equations (PDEs). We will investigate the origins of such equations (from, for example, physics), and their associated boundary and initial value problems. We will study how to solve such problems and various properties of solutions to such equations, all in a mathematically rigorous way. The overriding goals of the course are to enable you to:

- understand what mathematical questions related to a partial differential equation need to be answered;
- construct proofs which answer such questions and think logically;
- recall major results which can be useful in answering such questions (more complicated proofs need not be remembered but you should be able to understand them); and
- relate such statements to the physical situation they model;

Useful References: If you are looking for references for further reading, the following texts may be useful. You are not required to purchase any books, but obviously may if you so wish. Strauss contains most of the material we will study.

- W.A. Strauss, *Partial Differential Equations An Introduction*
- R. Shakarchi & E.M. Stein, *Fourier Analysis: An Introduction*
- L.W. Evans, *Partial Differential Equations*
- G.B. Folland, *Introduction to Partial Differential Equations*

Syllabus: The principal material in this course will roughly be the following.

- **The origins of PDEs, initial and boundary conditions, well-posedness.** Introductory examples, first-order linear equations, the method of characteristics
- **Waves and diffusion.** The wave equation, causality and energy, the heat equation.
- **Separation of variables.** Review of Fourier analysis, solving the heat and wave equations with Dirichlet, Neumann and Robin boundary conditions.
- **Harmonic functions.** Laplace's equation, Poisson's formula, Green's identities, the mean-value property, the maximum principle.
- **Numerical methods for PDEs.** Finite difference method, stability of numerical approximations, finite element method.
- **Nonlinear equations.** Calculus of variations, the Euler-Lagrange equation.

If there is time, we will cover additional material.

Seminars, Group Discussion and Homework: The course is composed of 18 seminars. Before all but the first seminar you are expected to prepare by reading assigned material and solving exercises. The time in seminars will be split between presentations, discussions and group work. In presentations the lecturer (and occasionally students) will summarise and elaborate upon material studied in the reading material. Discussions will be held with the whole class, often centred around possible

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solutions to exercises studied in small groups during the seminar. You do not need to prepare specifically for these in-class exercises, but the other preparation you do will even help you with these questions.

You are expected to play an active role in discussing exercises in class and may be asked to write up solutions to a small number of exercises.

Additional homework exercises will be assigned regularly to help you further understand and gain familiarity with the material. I am always happy to provide feedback on your written solutions to exercises. You are encouraged to work on problems together but make sure you really understand the material for yourself, as you will be taking the exam alone.

It is essential that you spent a significant amount of time thinking about the material taught and problems assigned in order to gain understanding of it and so pass the course. You are also welcome to ask me questions in or after class, or arrange an appointment at some other time if you would like additional help.

Assessment: There will be an examination later in the year. The University sets the date and time of the examination.

Academic Misconduct: Zero marks for the course will be assigned if evidence of cheating or academic misconduct is found in any part of your work for it. Please see the University guidelines on plagiarism and related matters.

David Rule