Graduate Course in Partial Differential Equations (MAI0133)
Spring Semester 2017

Homework 8 - Eigenvalues and Eigenfunctions

(Courant minimax principle) Let $L = -\sum_{i,j}^{n}(a^{ij}u_{x_i})_{x_j}$, where $((a^{ij}))$ is symmetric. Assume the operator L, with zero boundary conditions, has eigenvalues $0 < \lambda_1 \leq \lambda_2 \leq \cdots$. Show that

$$\lambda_k = \max_{S \in \Sigma_{k-1}} \min_{\substack{u \in S^\perp \\ \|u\|_{L^2=1}}} B[u,u] \quad (k = 1, 2, \dots).$$

Here Σ_{k-1} denotes the collection of $(k-1)$-dimensional subspaces of $H_0^1(U)$.