Homework 4: Intorduction to Conservation Laws

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Take the solution from Example 1 in Evans section 3.4

$$u(x,t) = \begin{cases} 1 & x \leq t,\ 0 \leq t \leq 1 \\ \frac{1-x}{1-t}, & t \leq x \leq 1,\ 0 \leq t \leq 1 \\ 0 & x \geq 1,\ 0 \leq t \leq 1 \end{cases}$$

and

$$u(x,t) = \begin{cases} 1, & x \leq s(t),\ t \geq 1 \\ 0 & x \geq s(t),\ t \geq 1 \end{cases},$$

where $s(t) = \frac{1+t}{2}$. We know that u satisfies that *Rankine-Hugoniot condition* (see Evans sec. 3.4). Show that u is an integral solution to

$$\begin{aligned} u_t + F(u)_x = 0,\ \text{in } \mathbb{R} \times (0,\infty) \\ u = g,\ \text{on } \mathbb{R} \times (t=0), \end{aligned}$$

i.e. show that u satisfies

$$\int_0^\infty uv_t + F(u)v_x dxdt + \int_{-\infty}^\infty gvdx\big|_{t=0} = 0.$$

The initial condition is

$$g(x) = \begin{cases} 1, & x \leq 0 \\ 1-x, & 0 \leq x \leq 1 \\ 0, & x \geq 1 \end{cases}.$$