Mathematical Methods for Social Scientists
Math 196 (Sec 55), Winter 2007
Revision Sheet for the Final Examination

The Final Examination will cover those sections in the text book which we have studied in class. Unlike the second mid-term no emphasis will be placed on any particular part of the course. The following questions are of the style you can expect in the exam.

1. (a) Given an $n \times n$ matrix A, define what it means for λ to be an eigenvalue of A with corresponding eigenvector $\mathbf{v}$. Define the eigenspace E_λ
   (b) Define the characteristic polynomial of a matrix A and what it means for A to be diagonalisable.
   (c) Show that the characteristic polynomial A of an $n \times n$ diagonalisable matrix A has the form $C_A(\lambda) = (\lambda - d_1)(\lambda - d_2)\ldots(\lambda - d_n)$
   (d) Compute all the eigenvalues and eigenspaces of the matrix

$$\begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}.$$

2. (a) Determine whether or not the matrix

$$\begin{pmatrix} 2 & 5 \\ 0 & 2 \end{pmatrix}$$

   is diagonalisable.
   (b) State a theorem about the linear independence of eigenvectors given information about their corresponding eigenvalues.
   (c) Show that λ is an eigenvalue if and only if $C_A(\lambda) = 0$.
   (d) Give an example of a 4×4 matrix which has as its only eigenvalues 0, 4 and π.

These questions concentrate on material covered since the second mid-term, since you do not have as many practice questions from those sections, but the final will cover the whole course. It will be useful to review your homework, previous revision sheets and previous exams, and make sure you understand them. I may also ask questions from the text book.