Name: ______________________________

Elementary Functions and Calculus III
Math 133 (Sec 22), Spring 2004
Mid-term 2
19th May 2004

Instructions: The total time allowed for this examination is 50 minutes. The use of notes, textbooks or calculators is prohibited. Write you answer to each question in the space provided below it. Should you require more space write on the reverse of the paper, labeling your answers clearly. Write your name at the top of this sheet. The maximum number of points available for each question or part question is shown in parentheses next to the question. There are 4 questions. You should attempt as many of the questions as you can. Partial credit may be given for incomplete answers.

(5 pts) 1. (a) State L'Hôpital's rule for indeterminate forms of the type ∞/∞.

(b) Use L'Hôpital's rule (for either forms of the type $0/0$ or ∞/∞) to evaluate the following limits.

(5 pts) i. $\lim_{x\to+\infty} \frac{\ln x}{x^{70}}$

(5 pts) ii. $\lim_{x\to+\infty} \frac{x}{e^x}$

(5 pts) iii. $\lim_{x\to 0} \frac{e^x - e^{-x}}{2\sin x}$

(7 pts) 2. (a) Consider a function $f : [a, \infty) \to \mathbf{R}$. Define the improper integral $\int_a^\infty f(x)\,dx$. When do we say such a thing converges or diverges?

(7 pts) (b) Use the definition above to calculate the following $\int_1^\infty \frac{1}{x^p}\,dx$ for $p > 1$.

(6 pts) (c) Prove whether or not the following improper integral converges or diverges: $\int_1^\infty \frac{1}{(x+1)^{\frac{3}{2}}}\,dx$.
(Hint: You don't need to compute its value.)

(6 pts) 3. (a) For a sequence $\{a_n\}_n$ define what it means to say the sequence converges to some limit l. What notation is used to denote this number?

(b) By using any method you know, show whether or not the sequence $\{a_n\}_n$ converges when a_n is defined as follows for all $n \in \mathbf{N}$.

(3 pts) i. $a_n = (\frac{n+1}{n})^n$,

(3 pts) ii. $a_n = \frac{60^k}{k!}$,

(3 pts) iii. $a_n = \sin(2\pi n)$.

(5 pts) (c) Suppose we knew that $a_n \geq L$ for some $L \in \mathbf{R}$ and for all $n \in \mathbf{N}$. What other fact would enable us to conclude that $\{a_n\}_n$ converged.

(7 pts) 4. (a) Define what we mean by saying $\sum_{n=1}^{\infty} a_n$ converges. What does it mean for this series to diverge?

(7 pts) (b) State the ratio test for a series. State carefully the required conditions.

(6 pts) (c) Use the ratio test to decide whether $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ diverges or converges.